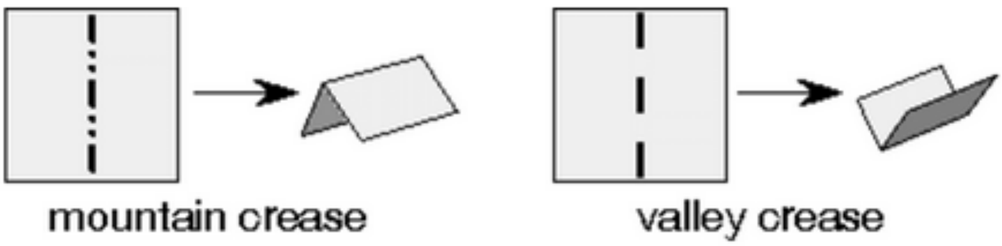
**THE MATHEMATICS OF PAPER FOLDING…**
**… in the context of STEAM – embedding the Arts in STEM**

Overview of Unit:
The mathematics of the Origami Sekkei (mathematical paper folding) project can be accessed by students from stage 1 to stage 5. The intention is to give emphasis to the movement from exploration of simple mathematical elements introduced in the early stages of mathematics, such as symmetry and transformations, through to the design and production of an industrial artefact. This journey from a blank, two dimensional shape to a three dimensional, functional object should reveal the more complex mathematics involved in achieving construction integrity.
For older students, the 2008 Ted presentation by Robert Lang is a good starting point. The final 9 minutes demonstrate the application of the mathematics of paper folding to science and medicine.
https://www.youtube.com/watch?v=NYKcOFQCeno

**STAGE 1**

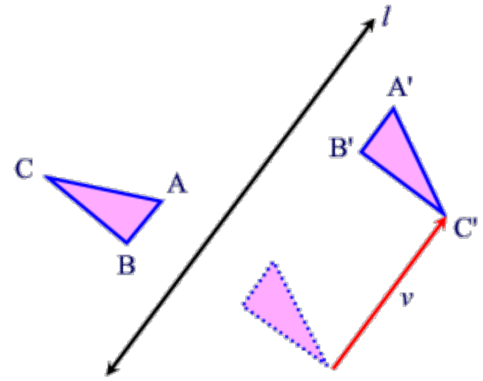
Syllabus Links:
Stage 1: Measurement and Geometry – Two dimensional Space. Experimenting with symmetry
MA1-1WM, MA1-3WM, MA1-15MG

Teaching:
Simple paper folding of squares and rectangles along lines of symmetry, noting the different geometrical shapes which are produced with each fold; experimenting with 'upwards folds' (mountains or hills) and 'downwards folds '(valleys); folding all the lines of symmetry or choosing some; discussing and experimenting with obtaining a flat to form folding. Some students Examples of symmetry in nature and art .
When given the sekkei templates students can identify the lines of symmetry, the up and down folds and which lines of symmetry have been used and those which have not. They may recognise how the patterns repeat.

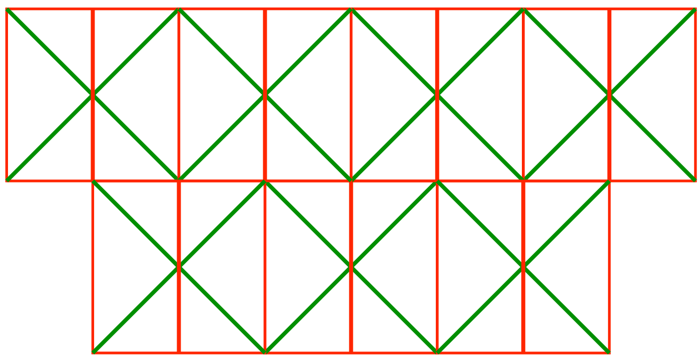


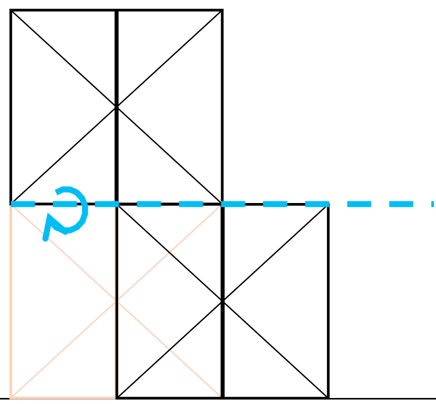

**STAGES 3 & 4**

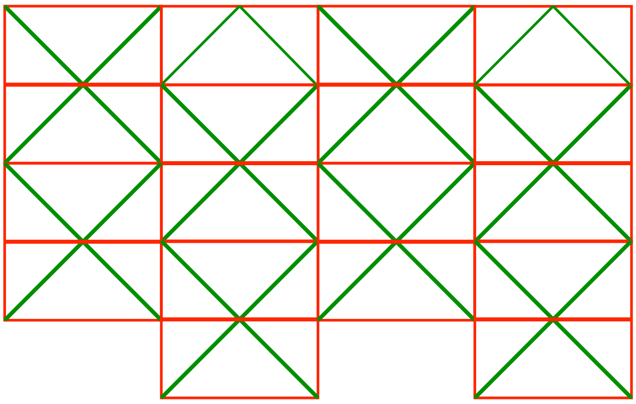
Syllabus Links:
Stage 3: Measurement and Geometry – Two dimensional Space – translations, rotations and reflections of two dimensional space.
MA3-1WM, MA3-2WM, MA3-3WM, MA3-15MG
Stage 4: Linear Relationships – transformations on the Cartesian Plane
MA4-1WM, MA4-3WM, MA4-11NA
Stage 4: Properties of Geometrical Figures 2 – recognising and defining congruent figures
MA4-1WM, MA4-2WM, MA4-3WM, MA4-17MG

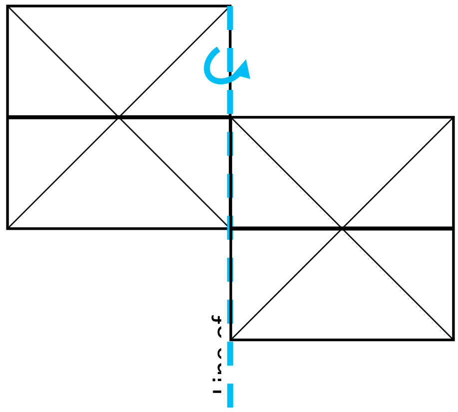



Teaching:
Symmetry of single geometrical shapes can be extended to recognising the symmetry of patterns that have been **transformed using rotation, translation, reflection and combinations of these.** A pattern is symmetric if there is at least one symmetry that leaves the pattern unchanged. By working by hand using grid paper and a single shape, or by creating the shape and symmetries using technology such as Geometers Sketchpad, students can experiment to discover the combinations of transformations which result in the symmetrical patters used in the sekkei templates.
In Stage 4, students work on the **Cartesian plane** with the original shape and use notation P etc to mark vertices in the original shape and P' and P'' to name the image resulting from each transformation. By investigation of the coordinates of each vertex in each transformation, the underlying definition of **plane symmetries** can be discovered, ie Plane symmetries (**rotation, translation and reflection)** move all points around in the plane so that their positions relative to each other remain the same although their absolute positions may change.
The notion of **congruency** can be introduced. Students investigate the properties of the basic geometrical shape and the geometrical shapes created by the internal lines of symmetry to confirm that each shape and its image is **exactly the same in terms of distances (side lengths) and angles**. A definition of congruency can be discussed and agreed.
**Congruent figures are a result of a rotation, translation or reflection, or a combination of these, of the original figure.**









DISCUSSION:

*The transformation used in the sekkei templates are glide reflections. A glide reflection is a combination of a line reflection and a translation along a line parallel to the line of reflection. Glide reflections are isometric, ie the original figure and the image are congruent (exactly the same).*

Identify the glide reflections in the lamps. Why have the basic shapes in the sekkei template been repeated using this combination of transformations? Why not a simple reflection?
(strength and rigidity)

Extension and Differentiation opportunities:
Vectors may be introduced to explain and illustrate transformations.
Symmetry in biology, chemistry and physics
Isometry and rigid motion

**STAGE 4**

Syllabus links:
Stage 4: Algebraic Techniques – generalises number properties to work with algebraic expressions
MA4-1WM, MA4-3WM, MA4-8NA

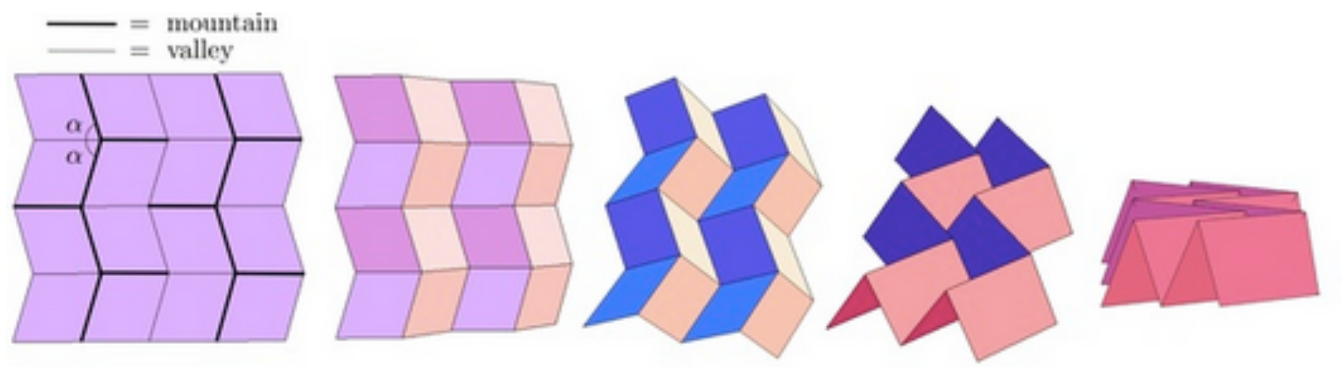
Teaching:
In this unit the students can discover the algebraic and angular properties of the origami sekkei folding. These, together with the pattern iteration using glide reflections, means that the lamps are constructed with strength and construction integrity but can be flat folded for storage, packaging, transport and delivery.

The real life problems which are being solved by flat folding involve situations where an item needs to be large at its destination but small for the journey, eg solar shields on space craft, a heart stent being 'delivered' through the blood stream. See the last 9 minutes of the Ted talk by Robert Lang https://www.youtube.com/watch?v=NYKcOFQCeno and a research group from Cornell university http://cohengroup.lassp.cornell.edu/research.php?project=10019.

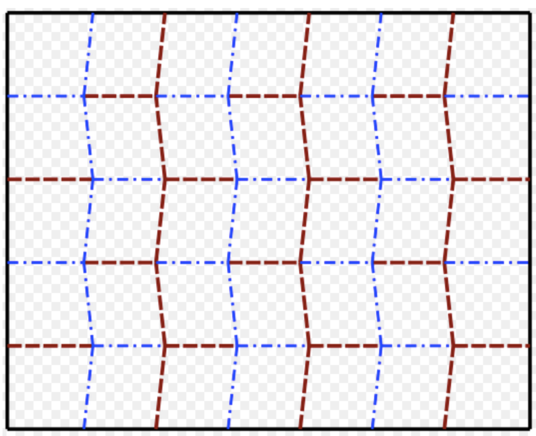
The algebraic and angular properties are Maekawa's theorem and Kawasaki's theorem, two of the essential properties of flat fold origami.

Maekawa's theorem states that at every vertex where creases interest, the difference between the number of mountain and valley creases is always 2. M-V=2 or V-M=2
Kawaasaki's theorem states that the sum of every other angle around a point is 180$^{o}$ (ie the sum of alternating angles).

First, have the students gain an understanding of what flat folding is by starting with a Miura map.




The Miura map fold crease pattern folds smoothly into a flat package. Tom Hull



The image can be reproduced on A3 paper (best) and the red lines and blue lines represent valley and hill foldings respectively. See https://en.wikipedia.org/wiki/Miura_fold for explanation and for a demonstration of how the end result flat folds.

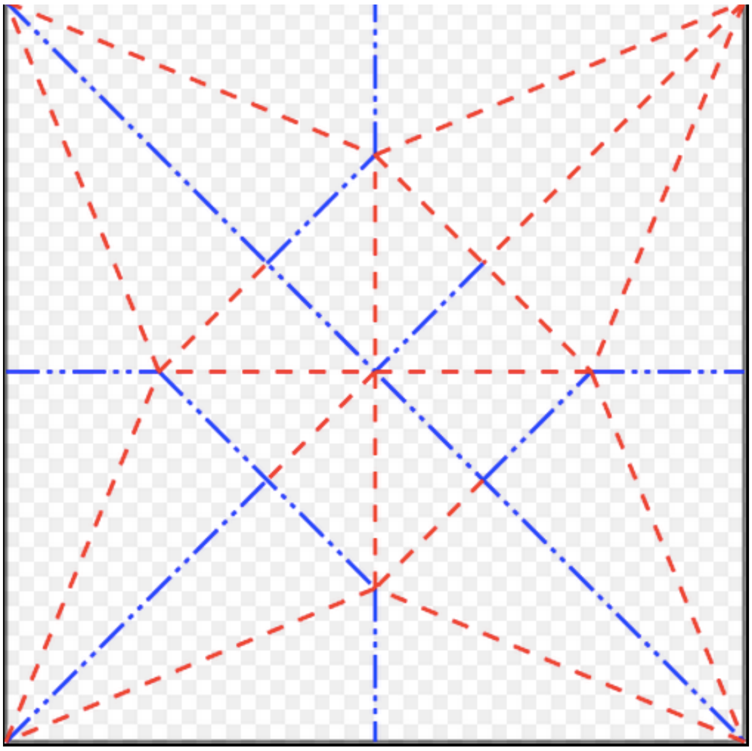
Once folded and demonstrated, students flatten out paper and mark up the valley and mountain folds in different colours.

In groups students can then examine the crease patterns of the lamps which have been folded and then flattened out. Folded and flattened out templates could be divided into 3 parts (cut up) for the purpose of the investigation. For each of these, the valley and mountain folds are marked in different colours. Once all have been completed, students can then be prompted to see if they can see a pattern around the vertices. The pattern can then be tested to see if it works by trying different numbers of valley and mountain folds.

A similar approach could be used to discover Kawaski's theorem concerning angles.

An alternative strategy for Maekawa's theorem could be to distribute templates, and one set of students given folding instructions and the other not.  This would be easiest done with a Miura map.  This would be a longer investigation, and being less guided, could lead to some interesting theories.

Both theorem's can be tested using this:



Extension:
Investigation applications of paper folding in science, technology and medicine; biomimicry and folding techniques.